

1.1

Focus on Problem Solving

Understand the Problem

Choose a Strategy

Carry Out the Strategy

Reflect

When you solve problems in mathematics, or in other subjects, a specific process helps you to organize your thoughts. This way, you can clearly understand the problem, devise a strategy, carry out the strategy, and reflect on the results.

Consider the following strategies when you are developing mathematical solutions to problems. You may use other strategies too.

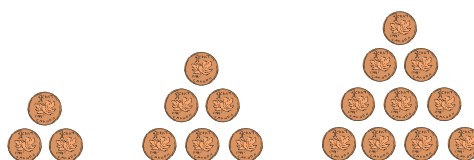
- Make an Organized List
- Look for a Pattern
- Work Backward
- Draw a Diagram
- Select a Tool
- Use Systematic Trial
- Use Logic or Reasoning

Investigate

How can a pattern help you solve a problem?

Part A: Make an organized list or table

Pennies are laid out in a triangular pattern as shown. How many pennies do you need to form a triangle with 10 pennies in its base?



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1. Read the problem above. Read it again. Express it in your own words.
2. A possible strategy is to identify and continue the pattern started in the diagram. Copy the diagram into your notebook.
3. Extend the pattern. Describe how the pattern develops. Use your description to extend it to a triangle with a base of 10 pennies. Record your numbers in a table with the following headings.

Diagram Number	Number of Pennies

4. Explain how you used the pattern to solve the problem. Can you find another pattern that could help you solve this problem?

Part B: Use Logic or Reasoning

In the grid, each letter represents a different **natural number** from 1 to 9. Use the clues to find the value of each letter.

- A, C, and G are **prime numbers**.
- A and I are both greater than 5.
- I is a multiple of H.
- B is $\frac{1}{2}$ of F, which is $\frac{1}{3}$ of E.
- C is greater than F.

A	B	C
D	E	F
G	H	I

1. Read the problem above. Read it again. Express the problem in your own words.

2. A possible strategy is to make an organized list or table. Write the numbers from 1 to 9 across the top of the table and the letters down the side.

	1	2	3	...
A				
B				
...				

3. Analyse the information given.

- Use the clues to mark Xs in the table for values that each letter cannot be. For example, A, C, and G are prime numbers, so they cannot be 1, 4, 6, 8, or 9.
- Put a check mark in your table as you confirm values of letters.

4. Verify that your results hold in the original grid.

natural number

- a number in the sequence 1, 2, 3, 4, ...

prime number

- a number with exactly two factors—itsself and 1

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Key Concepts

- Making an organized list or chart is a strategy that helps you to organize your thoughts and to see the information in an organized way.
- Identifying and describing a pattern is a strategy that can be used when a sequence of operations or diagrams occurs.
- When solving a problem, you will often use more than one strategy. Here are some problem solving strategies:
 - Draw a diagram.
 - Work backward.
 - Make a model.
 - Make an organized list.
 - Look for a pattern.
 - Find needed information.
 - Act it out.
 - Use systematic trial.
 - Use a formula.
 - Solve a similar but simpler problem.



Communicate Your Understanding

- C1** In the Investigate, Part A, the strategy recommended was to use an organized list.
- Which other strategies did you use?
 - How could you answer the problem using a different strategy?
- C2** In the Investigate, Part B, the strategy recommended was to make an organized list or chart. Which other strategies did you use?

Practise

- Continue each pattern for three more terms. Describe how to find successive terms.
 - 1, 3, 5, 7, 9
 - 4, 17, 30, 43
 - 2, 4, 8, 16
 - 1, 1, 2, 3, 5, 8
- You have two quarters, a dime, and a nickel. How many different sums of money can you make?
- Copy and complete these products.
$$1 \times 1 = ?$$
$$11 \times 11 = ?$$
$$111 \times 111 = ?$$
$$1111 \times 1111 = ?$$
 - Describe the pattern in the answers.
 - Use the pattern to predict the product $111\,111\,111 \times 111\,111\,111$.
- Evaluate each product.
$$11 \times 37 = ?$$
$$22 \times 37 = ?$$
$$33 \times 37 = ?$$
 - Continue and identify a pattern in the results.
 - Use the pattern to evaluate 99×37 .

Connect and Apply

In each problem, write one or two sentences to describe your strategy. Then, carry out your strategy.

- Express the fractions $\frac{1}{9}$, $\frac{2}{9}$, $\frac{3}{9}$, and so on as decimals. Describe the pattern.
 - How does the pattern change if the denominator is 99?
 - What if the denominator is 99 999?

6. A Sudoku is a Japanese number puzzle that follows a simple set of rules. Each three by three square, each row, and each column must contain each of the numerals 1 through 9 only once. Here is a Sudoku that is almost completed. What must the missing digits be?

4	8	9	5	3	2	6	1	7
2	6	1	8	7	9	5	4	3
7	5	3	4	1	6	9	8	2
6	7	8		4		1	5	9
1	3	2	9		8	4	7	6
5	9	4		6		2	3	8
8	2	5	7	9	4	3	6	1
9	4	6	3	8	1	7	2	5
3	1	7	6	2	5	8	9	4

7. Gina was born on September 15, 1997. Sam was born on January 23, 1994.
- How many years, months, and days old is each of them on January 1, 2020?
 - Describe a method of determining anyone's age in years, months, and days.
8. In the opening round of a chess tournament, players must play each other only once. How many matches are necessary in the opening round for a tournament that is set up for
- 2 players?
 - 3 players?
 - 4 players?
 - 10 players?
9. How many **perfect squares** divide evenly into 8820?

perfect square

- a number that can be expressed as the product of two identical factors
- 36 is a perfect square, since $36 = 6 \times 6$

Extend

10. In the following sum, each letter represents a different digit. F is half of C and $R = 7$. Find the value of each letter.

$$\begin{array}{r} \text{EAT} \\ + \text{FREE} \\ \hline \text{CAKE} \end{array}$$

11. In the grid, each letter represents a different natural number from 1 to 16. The sum of the numbers in each row, column, and diagonal is 34. Use these clues to find the value of each letter.

A	B	C	D
E	F	G	H
J	K	L	M
N	P	Q	R

- B, C, N, and R are greater than 12.
- H and D are perfect squares.
- R is double the value of J, which is double the value of D.
- Q is one third of F, which is half of E. G is 7 times A.
- P is less than or equal to 4.